Problem Solving

What Kinds of Problems?

Supplement #3 to *The Scots of Scarborough* V1, #1

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Simplification or Modeling

- A key notion in problem-solving is to first simplify the problem such that the problem is more understandable
- Making reasonable / valid assumptions is typically required
- Break the large problem into smaller problems
- What do I already know that is relevant?
- Translate the problem statement into a more convenient form (e.g. translate a list of numbers into a graph)
- Simplification requires core thinking skills
- A “Model” is a “Simplification” of something more complex

Several Familiar Models

Complex Issue	Simplification / Model
World of Work	10 Broad Based Technologies
Technology – What to Learn	13 Fundamental Concepts
How to Learn	6 Learning Skills
How to Get Marks	4 Achievement Categories
Good Attitude	10 Character Traits

… Similarly, Problem Solving Itself is a Complex Issue

- Just as the huge issue of “How to Learn” can be broken down into six “Learning Skills” … which you will “tackle” one at a time, bit by bit, coming to a better understanding of each skill or strategy as you go…
- The huge issue of Problem Solving can be broken down into five “Types of Problems” beginning with simple problems and progressing to more difficult, complex problems. First master Type 1, then Type 2…
- Each “Type” of problem involves learning about some new concepts and practising with new tools / strategies
- One simple problem-solving strategy: if you can’t yet “see” the problem – re-represent it, ie look at it in another way such as by translating the data into a graph

But… a Word of Caution

- In general, problem-solving involves abstracting the situation – modeling and simplifying
- We simplify the situation in order to better understand it and to enable certain techniques and strategies more likely to render a sensible result
- The more we simplify a situation, the less “correct” a particular “solution” might be
- Situation over-simplification in history class, for example, can lead to gross errors

An Engineer’s Mantra

“*When there is doubt, get more data.*”

- But… sometimes, getting more data could make the overall system more complex and, hence, more difficult to understand
- Real-world problem-solving is a constant compromise between model complexity and solution viability
- Engineers use their professional judgement and experience to decide just when they have reached an acceptable compromise
- They also apply a “Factor of Safety” to their design work… to be discussed later!

Five General Types of Problems to Solve

1. ***Simple Operation***: Single Step (common in math class; eg. Do I multiply or divide?)
2. ***Exploration***: Multiple Step / Linear System (common in math class)
3. ***Process***: Inquiry-Based (often connecting math and science or math and history etc.)
4. ***Design***: Open-Ended Data-Gathering (often bridging math, science and tech studies)
5. ***Fabrication***: Open-Ended Inquiry-Design-Build-Assess (eg. Technological Design)

Type 1 Problems: Simple, Single Step

- One decision needs to be made
- Identify and use the appropriate operation (for example, in math, add, subtract, multiply, divide, square)
- Recognize symmetry, equivalence etc.
- Translation of information into a more convenient form is a helpful technique
- Just as for more complex problems, success depends on students having certain prerequisite knowledge of both concepts and procedures
- In Math, there is always a single correct answer after making reasonable assumptions regarding the system and precision

Problem Solving Activities: Type 1

- Examples of Type 1 Problems (single step) include:
  - Do I multiply or divide?
  - Do I add or subtract?
  - Situations involving yes / no possibilities including ‘on’ vs. ‘off’, ‘high’ vs. ‘low’, ‘left’ vs. ‘right’ and other “opposites”; equality, inequality
  - Many Type 1 problems can be given to students as True / False and Multiple Choice questions.
  - To “Show all work” adds value to the learning -- using sound process is very important

Type 1 Sample Problem

Tom has 2 tons of scrap steel. Bill at the junk yard says he'll give him $250 per ton. How much money will Tom get in exchange for his scrap steel? Show all work.

Multiply is the operation to be used in a simple equivalent arithmetic statement.

2 tons x 250$/ton = 500$

Note the placement of the units. “Tons” cancel leaving $ as the units in the solution. Cancelling units is a very powerful problem-solving technique. It can also be used to validate an answer.

Type 2 Problems: Exploration / Multiple Step

- More than one sub-problem
- Interpret the problem situation and break it down into the most appropriate sub-problems
- Then translate each sub-problem into equivalent, simpler and more understandable representations or models
- Make reasonable assumptions regarding the system and precision. (This is an area in which this problem type can border on Type 3.)
- There is typically a single correct answer in math class once a decision is made about the required precision
- Students should “show their work” thus demonstrating solution planning skills

Type 2 Sample Problem

Tom has a machine that has 4020 pounds of steel in it. Bill at the junk yard says he'll give him $250 per ton. But Tom must pay the $9.50 recycling fee for each of the 4 tires on the machine. Tom also has 100 pounds of scrap lumber which Bill does not want. How much money will Tom get in exchange for his scrap? Show all work.

Solution Requires More Organized Work

Steps and Sub-problems:

1. Filter out irrelevant information / data
2. State assumptions
3. Make a decision on converting decimals into fractions
4. Recall what a ton is in pounds (could be given in the problem statement, but not necessarily – finding information is another thinking skill)
5. Translate the problem statement into a consistent sets of units (tons or pounds)
6. Break the problem down into sub-problems
    1. Multiply to get dollar value of the steel **
    2. Multiply to get the dollar value of the tire recycling fee **
7. Subtract

**** NOTE: Translate the sub-problems into the correct algebraic statements, including units, cancel the units as you would reduce a fraction and then apply the BEDMAS order of operations.**

Type 3 Problems: Analysis / Inquiry

- Deeper analysis is required for greater understanding of the nature of the problem -- simplify / abstract the problem
- Be more creative in breaking down the problem into sub-problems. All assumptions should be reasonably valid.
- Create an alternative and appropriate model or models of the problem system
- Translate (or re-represent) the relevant data into a more useful form (or forms) such as a table or graph
- Adapt other ways of viewing how the events in a problem situation happen
- May need to research scientific or social concepts to achieve the required minimum level of understanding of the system
- Not always a single correct answer. There may be a range of possible answers, several of which are very reasonable and one is perhaps "best".

Type 3 Sample Problem

Tom has an old agricultural machine – a tractor -- from 1921 that is made of steel, copper, maple wood and synthetic rubber that weighs 2050 pounds. The weights of each type of material in the machine are shown in the table which the teacher will provide. Bill at the junk yard says he'll give him the dollar amounts for each type of material shown in the bar graph which the teacher will provide. Tom must pay the $9.50 recycling fee for each of the 4 tires on the machine. The old machine is an antique. The value of this type of antique depends on its age as shown in the graph given by the linear equation which the teacher will provide. What should Tom do? Show all work.

Solution Requires More Decision Making

- Assumptions are typically necessary, indeed important.
- This problem involves decision-making to convert data into forms that are easy to perform operations on
- Multiple sets of input information are provided – or need to be otherwise obtained
- Multiple calculations are required
- Comparison is required
- A linear system is often involved in the comparison
- This problem does not clearly state: "*What should Tom do in order to get the most money for his machine?*" – but note that this ***could be*** assumed.
- There's more than just Math. The inquisitive student should take the initiative to investigate the intrinsic value of preserving a piece of our agricultural heritage.

Type 4 Problems: Synthesis / Design

- Open-ended data gathering problem
- This is getting closer to real world problem-solving
- Student creates own problem statement or, more generally, interprets and then verifies what his / her customer requested
- Make very significant decisions that could affect the success or failure of the customer's project
- This type of problem very strongly exercises student's Analysis and Synthesis thinking skills (analysis of the situation and then synthesis of a solution strategy)

Type 4, Continued (2)

- Type 4 problems often combine several disciplines in creative ways so that students will see that life is not composed of isolated vertical silos -- science, math, history etc. all off by themselves.
- Life's problems are a complex intertwining of many disciplines or subject areas.

Type 4, Continued (3)

- Student must measure something and synthesize a "plan for how to do it":
  - What to measure: the quantity or parameter
  - How to measure it: what units; accuracy required; instrumentation selection
  - When and where to measure it: does temperature / humidity affect it?
  - Why to measure it: what is the goal of the measurement program?
  - Who to measure it: skills / qualifications required; knowledge gaps that should first be filled
  - Propose a detailed solution strategy on paper
  - Conduct all measurements, record observations, validate results and state conclusions

Type 4 Sample Problem

- Same problem as Type 3 above with the following exceptions:
  - Many sets of data regarding the price of scrap materials over time are given, but the most recent data is 4 years old. Some of the data may not be uniformly linear over time.
  - No graph of antique machine values is given -- but names and phone numbers of 5 antique dealers are given.

Solution Requires More Decision Making

- Extrapolate to determine likely scrap prices today -- must decide whether the long-term slope of the relationship is more reasonable than the most recent slope of the scrap price relationships
- Design a protocol to determine the antique value of this machine - must be sure to evaluate the dealer's qualifications in this area of antiques. If the 5 dealers give a range of possible values, students can calculate an average for example.
- The ethical aspect of the problem remains

Type 5 Problems: Fabrication

- Open-ended inquiry / design / build / assess problem
- This is closest to real world problem-solving – includes all of the features of Types 3 and 4 (Analysis and Synthesis)
- Decision-making is a key element – and be sure to record the rationale you used in your decision-making. Engineers may need the rationale in court!

Type 5, Continued (2)

- Tool-building and / or product building is the essence of this problem type
- Student may also build a version 1 prototype tool for the purposes of solving the overall problem, for example:
  - build a database for fact-finding survey data
  - build a solar energy concentrator
- Sometimes multiple tools are required
- May design new processes to follow
- Sometimes an entire product is the solution to the problem, subject to testing and validation.

Type 5 Sample Problem

- Same problem as Type 3 above with the following exceptions:
  - Your class has requested the old machine for the purposes of "breaking it apart" so that some of its many parts can be "re-configured" into marketable items for fundraising
  - The statement "What should Tom do?" is changed to "*What should your class and Tom do? Explain in detail. Provide all design process (planning) documents (eg Design Brief, Requirements, Research, Specifications etc.)*."

Solution Requires More Decision Making

- Completely analyze the old farm machine to determine the nature and quantity of materials and parts. Reverse-engineer some of its key parts to see '*what makes it tick*'.
- Design marketable (eg novelty) items using various machine parts
- Calculate costs of other materials that are required in order to make the parts marketable
- Build the marketable products, then test them
- Market the products

Solution Requires More Decision Making (2)

- The mathematics can involve costs of processing the various parts and the revenue derived from sales.
  - The students design the "math problems" based on the parts in the machine and the purposes they perceive for them.
- The ethical decision-making remains, but with the added elements of business opportunity and hands-on design and building

Reference Materials

- *Growing Success: Assessment, Evaluation and Reporting in Ontario Schools*, Queen's Printer for Ontario, 2010.
- R. Case and L. Daniels, Preconceptions of Critical Thinking from a partial draft of *Tools for Thought*.
- R. Charles, F. Lester, P. O'Daffer, *How to Evaluate Progress in Problem-Solving*, National Council of Teachers of Mathematics, 1987

Suggested Exercises – Type 1

1. You determine that each of your four bridge towers will require 24 linear feet of 2x4 maple bracing. How many feet of 2x4 bracing will your bridge towers need in total? Include the units in the algebraic expression and cancel the units just as you would reduce a fraction.

Suggested Exercises – Type 2

2. Using the 7-step process on slide 13 above, solve the Type 2 problem as stated on slide 12. Be sure to include the units in your algebraic statements and then cancel the units just as you would reduce a fraction.

Suggested Exercises – Type 2

3. Now expand the 2x4 problem on slide 28 to a Type 2 problem. For example, add this statement: *"If a 2x4 stud comes in 8 foot lengths, how many studs must you buy?"* (There is an important assumption to make in the solution to this Type 2 problem -- discuss.)
   – **Note**: Create your own Type 2 "2x4" problem, starting with the Type 1 on slide 28.

Suggested Exercises – Type 3

4. Consider the Sample Type 3 problem statement on slide 15. Work in teams of 3. The supply teacher does not have the 3 supporting handouts – but you are keen to learn!
   - Student #1 will make an assumption regarding the weights of each type of material in the machine.
   - Student #2 will make an assumption regarding the dollar amounts for each type of material.
   - Student #3 will make an assumption regarding the Value / Age linear relationship.
   - Each student will translate his / her assumption into a form that is most useful in terms of solving the overall problem.
   - Now, each of you will solve the overall problem independently of one another (but using all three of the assumptions). Show all work, cancel units of measure in your algebraic statements.
   - Compare / contrast your 3 solutions. Discuss your process. Discuss your decision-making.
   - Explore some “what-ifs” – “What if we assumed this… or this…?”

Suggested Exercises

5. Compare and contrast the Five Types of Problems vs the Seven Core Thinking Skills. Which Core Thinking Skills will tend to predominate in finding solutions to each Type of Problem?
6. Consider this statement: *“A Type 5 Problem only requires the use of the 3 highest thinking skills*”. Do you agree or disagree? Explain your reasoning. Give examples supporting your position.